

# C.U.SHAH UNIVERSITY

## Summer Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT3

Branch: B.Tech (All)

Semester : 2

Date : 25/04/2018

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

a) The infinite series  $1 + r + r^2 + \dots + r^{n-1} + \dots$  is convergent if

(A)  $|r| < 1$  (B)  $|r| > 1$  (C)  $r = 1$  (D)  $r < -1$

b) The sum of the series  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$  is

(A)  $\log 2$  (B) zero (C) infinite (D) none of these

c) The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^7 x \, dx$  is

(A)  $\frac{32\pi}{35}$  (B)  $\frac{32}{35}$  (C) zero (D)  $\frac{16}{35}$

d) If  $f_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$ , then  $(f_n + f_{n-2})$  is equal to \_\_\_\_\_.

(A)  $\frac{1}{n}$  (B)  $\frac{1}{n-1}$  (C)  $\frac{n}{n-1}$  (D)  $\frac{n-1}{n}$

e)  $\int_1^{\infty} \frac{1}{x^{\sqrt{2}}} \, dx$  is convergent.

(A) True (B) False

f)  $\sqrt[n]{n} \sqrt[n-1]{n-1} =$  \_\_\_\_\_

(A)  $\frac{\pi}{\cos n\pi}$  (B)  $\frac{\pi}{\sec n\pi}$  (C)  $\frac{\pi}{\cos ecn\pi}$  (D)  $\frac{\pi}{\sin n\pi}$

g) If  $B(x, 2) = \frac{1}{3}$ , then the value of  $x =$  \_\_\_\_\_.

(A) 0 (B) 1 (C) 2 (D) none of these

h) If the power of  $y$  are even, then the curve is symmetrical about

(A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these



- i)  $\int_0^1 dx \int_0^x e^x dy$  is equal to  
 (A)  $e+1$  (B)  $e-1$  (C)  $\frac{1}{2}(e+1)$  (D)  $\frac{1}{2}(e-1)$
- j) On converting into polar coordinates  $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} dx dy$  is equal to  
 (A)  $\int_0^{\pi} \int_0^{2a\cos\theta} r dr d\theta$  (B)  $\int_0^{\frac{\pi}{2}} \int_0^{2a\cos\theta} r dr d\theta$  (C)  $\int_0^{\frac{\pi}{2}} \int_0^{2a\sin\theta} r dr d\theta$  (D) none of these
- k) The transformations  $x+y=u, y=uv$  transform the area element  $dy dx$  into  $|J| du dv$ , where  $|J|$  is equal to  
 (A) 1 (B)  $u$  (C)  $-1$  (D) none of these
- l) The degree of the differential equation  $3 \frac{d^2 y}{dx^2} = \left\{ 1 + \left( \frac{dy}{dx} \right)^2 \right\}^{\frac{3}{2}}$  is  
 (A) 1 (B) 2 (C) 3 (D) 6
- m) The solution of  $\frac{dy}{dx} = e^{x+y}$  is  
 (A)  $e^x - e^{-y} = c$  (B)  $e^x - e^y = c$  (C)  $e^x + e^{-y} = c$  (D)  $e^x + e^y = c$
- n) The orthogonal trajectories of the family of curve  $y = cx^k$  are given by  
 (A)  $x^2 + ky^2 = \text{const.}$  (B)  $x^2 + cy^2 = \text{const.}$  (C)  $kx^2 + y^2 = \text{const.}$   
 (D)  $x^2 - ky^2 = \text{const.}$

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

a) Test the convergence of the series  $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$  (5)

b) Using reduction formula evaluate:  $\int_0^1 x^6 \sin^{-1} x dx$  (5)

c) Prove that  $\int_0^{\infty} \frac{x^4 (1+x^5)}{(1+x)^{15}} dx = \frac{1}{5005}$ . (4)

**Q-3 Attempt all questions (14)**

a) Prove that  $\int_0^1 x^{q-1} \left( \log \frac{1}{x} \right)^{p-1} dx = \frac{\Gamma(p)}{q^p}$ . (5)

b) Using reduction formula prove that  $\int_0^a x^5 (2a^2 - x^2)^{-3} dx = \frac{1}{2} \left( \log 2 - \frac{1}{2} \right)$ . (5)

c) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$ . (4)

**Q-4 Attempt all questions (14)**



a) Change the order of integration in the integral  $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$  and evaluate it. (5)

b) Examine the series  $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots + \frac{x^n}{n^2 + 1} + \dots$  for convergence using ratio test. (5)

c) Solve:  $(y^2 e^{xy^2} + 4x^3) dx + (2xy e^{xy^2} - 3y^2) dy = 0$  (4)

**Q-5**

**Attempt all questions (14)**

a) Solve:  $\frac{dy}{dx} = 2y \tan x + y^2 \tan^2 x$  (5)

b) By changing into polar co-ordinates, evaluate the integral (5)

$$\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx dy .$$

c) Using reduction formula, evaluate:  $\int_0^{\infty} \frac{x^4}{(1+x^2)^4} dx$  (4)

**Q-6**

**Attempt all questions (14)**

a) Evaluate:  $\int_0^{\infty} x^4 e^{-x^4} dx$  (5)

b) Solve:  $xdy - ydx = \sqrt{x^2 + y^2} dx$  (5)

c) Evaluate:  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$  (4)

**Q-7**

**Attempt all questions (14)**

a) Trace the curve  $r^2 = a^2 \cos 2\theta$ . (5)

b) Evaluate:  $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$  (5)

c) Find the length of the arc of the curve  $y = \log \sec x$  from  $x=0$  to  $x = \frac{\pi}{3}$ . (4)

**Q-8**

**Attempt all questions (14)**

a) Show that the volume of the spindle-shaped solid generated by revolving the (5)

astroid  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$  about the x-axis is  $\frac{32\pi a^3}{105}$ .

b) Trace the curve  $y^2(2a-x) = x^3$ . (5)

c) Investigate the convergence of  $\int_2^5 \frac{1}{\sqrt{(x-2)}} dx$ . (4)

